The Interfacial Drag and the Height of the Wall Layer in Annular Flows

A method to predict the height of the wall layer and the interfacial drag in annular flow under conditions that the flow rate of the entrained liquid is known is developed from measurements on air-water flow in circular tubes. Flow conditions are found to be characterized by a generalized Martinelli flow parameter.

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SCOPE

In the annular regime observed for gas-liquid transport in a pipeline, a liquid layer flows along the wall and a high velocity gas stream flows concurrently. The liquid layer has an agitated wavy surface, and it can be entrained into the gas. This entrained liquid is carried along as droplets with a large range of diameters. The roughened interface causes an increase in the drag of the gas on the liquid τ_i and, consequently, a larger frictional pressure loss than would exist if the gas were flowing in a smooth walled channel.

Hewitt and Hall-Taylor (1970) have recently shown how a knowledge of τ_i, the time averaged height of the wall layer m, and the fraction of the liquid entrained $E = W_{LE}/W_L$ play central roles in the development of improved design relations for systems operating in this flow regime. This paper presents correlations for τ_i and m under conditions that E is known. The principal contributions are that direct relations with controlled variables are developed and that a comprehensive review of available measurements is undertaken. The chief limitation is that the correlation is based entirely on measurements with the air-water system.

CONCLUSIONS AND SIGNIFICANCE

Two dimensionless groups are found to be of primary importance. One of these, F, which characterizes the flow condition, is quite similar to the Martinelli flow parameter. The other, G, is a measure of the importance of gravity g relative to the gas drag τ_i.

The friction factor for annular flows f_i is found to increase linearly with F for vertical upflows:

$$\frac{f_i}{f_s} = 1 + 1\,400F$$

where $f_s = 0.046\ Re_G^{-0.20}$ is the friction factor for a smooth walled tube. For vertical downflows, the gravity G group can also be of importance, and we recommend either the use of the following equation

$$\frac{f_i}{f_s} = 1 + 1\,400F\left[1 - \exp\left(-\left|\frac{\tau_i}{\rho_L g m}\right|\right)\right]$$

or Equation (45) to approximate the term $\tau_i/\rho_L g m$. For horizontal flows, we base the friction factor f_i on the average shear stress τ_i around the circumference. From the very limited amount of data available, we tentatively recommend the following equation:

$$\frac{f_i}{f_s} = 1 + 850F$$

For vertical upflows and downflows, we find that the ratio of the height of the wall layer to the hydraulic diameter is given as

$$\frac{m}{d_t} = \frac{6.59F}{(1 + 1\,400F)^{1/2}}$$

For horizontal flows, we define m as the average height of the wall layer around the circumference. Data available for horizontal flows are not sufficient to establish an accurate relation for m. However, they indicate that the equation for vertical flows underpredicts m/d_t, so we tentatively suggest that $(m/d_t) = (6.59F)/(1 + 850F)^{1/2}$.

Presently available direct correlations of m with controlled variables use measurements of the void fraction β and the assumption that all of the liquid is flowing in the wall layer; that is, the equation

$$\beta = \frac{4m}{dt} \tag{1}$$

For example, from the correlation for β developed by Martinelli and his co-workers from data on small diameter horizontal pipelines (Butterworth, 1974), the following equation is obtained for the height of the wall layer:

$$\frac{4m}{d_t} \Big/ \left(1 - \frac{4m}{d_t}\right) = 0.28X^{0.71} \tag{2}$$

Here X is the Martinelli flow parameter which is a function of the ratio of the liquid and gas viscosities and densities and the gas Reynolds number. For $Re_L < 1\,000$, $Re_G > 1\,000$

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$$X = X_{vt} = 16.9\left(\frac{\mu_L}{\mu_G}\right)^{0.5}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}\left(\frac{W_L}{W_G}\right)^{0.5} Re_G{}^{-0.4} \quad (3)$$

and for $Re_L > 1\,000$, $Re_G > 1\,000$

$$X = X_{tt} = \left(\frac{\mu_L}{\mu_G}\right)^{0.1}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}\left(\frac{W_L}{W_G}\right)^{0.9} \quad (4)$$

The Reynolds numbers used to characterize the flow of the liquid and the gas in the above equations are calculated as if the liquid or gas were flowing alone in the conduit; that is

$$Re_L = \frac{d_t W_L}{A_t \mu_L} \quad \text{and} \quad Re_G = \frac{d_t W_G}{A_t \mu_G}$$

Hewitt and Hall-Taylor (1970) find that the Martinelli correlation can overpredict the liquid volume fraction by as much as 100% for upward annular flow of air and water in a 3 cm tube. Analyses presented by Hewitt and Hall-Taylor (1970) indicate that a void fraction correlation of the form $\beta/(1 - \beta) = f(X)$, suggested by Martinelli, implies that all of the liquid is flowing as a layer on the wall. The overprediction of the liquid fraction and consequently the wall layer height by (2) could then be explained because some of the liquid is flowing as entrained droplets. Their analysis suggests that a better correlation for m might be obtained if X is calculated from the liquid flow rate in the wall layer W_{LF} rather than the total liquid flow rate W_L.

One of the first analytical treatments of the effect of an adjoining gas flow was performed by Semenov (1944). He attempted to calculate the onset of flooding as a function of interfacial shear due to a concurrent or countercurrent gas stream. His results are only applicable to a laminar liquid film in the absence of waves.

More recent correlations for the height of the wall layer have involved the development of complicated triangular relations among m, τ_i, and E as outlined in the book by Hewitt and Hall-Taylor (1970). The basic assumptions used in these works are that the variation of the velocity, mixing length, or eddy viscosity in the wall layer is the same as for a single-phase flow and that the interfacial stress is strongly related to flow characteristics of the wall layer.

Hoogendoorn and Welling (1965) calculated heights of the wall layer assuming rectilinear laminar flow. Anderson and Mantzouranis (1960) did a similar analysis but used the universal velocity relation developed for turbulent flow close to a wall. These works implicitly assumed a constant shear stress in the liquid equal to τ_i. Levy (1966) recognized the importance of the variation in shear stress but then neglected it in performing an analysis similar to that of Anderson and Mantzouranis. Shearer and Nedderman (1965) used the actual shear distribution in the momentum equation but integrated it only for the laminar flow case. Collier and Hewitt (1961) suggested that the effect could be allowed for by use of a correction factor derived from a consideration of sheared laminar flow.

The most systematic approach to calculation was presented by Dukler (1959). In considering downward flow, he used the actual shear stress distribution and a two region eddy viscosity model in the momentum equation. This was integrated numerically in one region and analytically in another to give a dimensionless film height as a function of film Reynolds number and a parameter related to shear variation in the film. Results were presented graphically. Later, Hewitt (1961) performed the corresponding analysis for upward flow, pointing out a mathematical error in Dukler's work. His results were presented tabularly.

The interfacial drag τ_i is directly related to the frictional pressure gradient through the force balance

$$\tau_i = -\frac{(d_t - 2m)}{4}\frac{dp_F}{dz} \quad (5)$$

where d_t is the tube diameter. Consequently, τ_i can be calculated from empirical relations for dp_F/dz for two-phase flows such as developed by Martinelli and his co-workers. Recently, it has been suggested that more accurate relations for τ_i for annular flow can be obtained by directly taking into account some of the features of the phase distribution that exist in this regime.

Calvert and Williams (1955) argued that the projected area for drag caused by the wavy interface could be characterized by the thickness of the wall layer. They presented an empirical dimensional relation which showed an increase of the drag coefficient with increasing m. Levy (1966), and later Wallis (1969), presented correlations of interfacial friction factor with the ratio of film thickness to pipe diameter. Levy argued that the shear stress in the liquid could be estimated by an ordinary mixing length model for which the magnitude of the mixing length would depend only on the distance from the solid wall. Anderson and Mantzouranis (1960) presented friction factors correlated with the height of the wall layer made dimensionless with all parameters rather than the pipe diameter. Shearer and Nedderman (1965) and Gill et al. (1963) nondimensionalized the height in the same way but used an effective roughnesses, analogous to Nikuradse's sand roughnesses, instead of a friction factor.

In this paper, an approach similar to that of Dukler (1959) and of Hewitt (1961) is used to develop a relation for the film height. An equation for the velocity distribution in the wall layer is obtained from the macroscopic force balance equation by using the van Driest mixing length model. This is integrated to give the height as a function of the volumetric flow. By using an appropriately defined average shear stress, rather than τ_i, to characterize the stress, a correlation is developed which is not very sensitive to shear stress variation in the liquid layer. Data on τ_i are shown to correlate with a combination of controlled variables suggested by the analysis for the height of the wall layer and the observation that pressure drop is strongly dependent on this height. This approach is found to be superior to the correlations discussed by Hewitt and Hall-Taylor (1970) that relate τ_i directly to m, since m itself is strongly dependent on τ_i. An equation for the height of the wall layer in terms of controlled variables is obtained by eliminating τ_i from the height relation derived from the momentum balance equation. These final relations of τ_i and m to controlled variables developed from recent theories are found to be quite similar to those suggested from the earlier work of Martinelli and his co-workers.

DATA USED TO DEVELOP AND TEST THE CORRELATIONS

The relation developed for the height of the wall layer was tested with height measurements of free falling layers and with gas-liquid flow experiments in vertical tubes for which the height, frictional pressure drop, and entrainment were measured. The gas-liquid flow experi-

TABLE 1. DATA ON PRESSURE DROP AND FILM HEIGHT FOR AIR-WATER SYSTEMS

	Re_G	Re_{LF}	D(cm)	Flow direction	Key to figures
Willis (1965).	5 000-25 000	230-1 280	1.28	Up	+
Whalley, et al. (1973)	17 000-159 000	350-3 450	3.18	Up	⦶
Gill, et al. (1963)	25 000-104 000	150-4 300	3.18	Up	⊟
Collier and Hewitt (1961)	75 000-164 000	35-335	3.45	Up	△
Chien and Ibele (1964)	25 000-112 000	1 150-15 100	5.08	Down	×
Charvonia (1959)	52 000-255 000	20-1 200	6.35	Down	◇
Wright and Laufenbrenner (1975)	5 000-33 000	850-7 700	2.54	Down	△
Butterworth (1973)	56 000-112 000	3 750-9 250	3.18	Horizontal	⊠
Swanson (1966)	40 000-100 000	1 000-9 100	2.54	Horizontal	⊗

ments were also used to develop the relation for the interfacial stress. The interpretation of experiments with horizontal flows is more complicated than with vertical flows, since the height of the wall layer and the stress at the interface depend on the location on the tube wall. No attempt was made to predict this circumferential variation. The correlations developed from the vertical flow data were tested only with respect to their ability to predict a circumferentially averaged height and interfacial stress. The range of variables covered in the experiments is summarized in Table 1.

Measurements for gas-liquid upflows were obtained from the studies of the United Kingdom Atomic Energy Authority at Harwell with various amounts of entrainment and from a paper by Willis (1965). The Harwell group measured the height of the wall layer with a conductance technique. Willis was able to use holdup measurements to calculate m, since entrainment was negligible in his experiments. Charvonia (1959), who studied gas-liquid downflows, measured m by light absorption. He reported negligible entrainment. Chien and Ibele (1964) did a similar study. They did not measure entrainment but reported the point at which it began to be significant. Only data for which it was not important were used. Wright (1975) worked in downflow with no entrainment and measured m by a conductance method.

Considerable data on the height of the wall layer and on pressure drop were published by the CISE Laboratory (Alia, 1965), but, since entrainment was not measured, they were not used. This is unfortunate, since fluid pairs other than air-water were investigated. Webb (1970) worked with downflow in the same equipment used for the upflow experiments at Harwell. He reported all pertinent variables, but his pressure drop measurements appear to be in error. They are systematically lower than measurements made in other laboratories under the same conditions. Some of his results are well below the expected values for a smooth tube. Chu (1973) also reported all variables of interest, but in his case the wall layer height measurements appear to be in error, since his results for free falling films are considerably lower than the values calculated from established relations for turbulent conditions. The experiments in horizontal flows by Butterworth (1973, 1974) and by Swanson (1966) reported the thicknesses of the wall layer at several points around the circumference. These were used by simply taking an arithmetic average. Interfacial stresses were calculated from their measured pressure drops as though they did not vary with circumferential position. Both workers used a conductance method to measure film heights. The correlations developed by Brauer (1956) and by Feind (1960) from free falling turbulent wall layers were also used. Brauer used a probe that made electrical contact with the liquid to determine m; Feind used a drainage method with various fluids whose kinematic viscosities varied from 1 to 19.7 centistokes.

RELATION FOR FILM HEIGHT

The General Solution

The distribution of shear stress τ for a fully developed wall layer in a vertical tube is obtained by a simple force balance:

$$\tau = \frac{1}{r}\left(R\tau_W - \frac{\rho_L \tilde{g} R^2}{2}\right) + \frac{\rho_L \tilde{g}}{2} r \qquad (6)$$

The quantity $\tilde{g}$ includes the combined influence of gravity and the pressure gradient on a fluid element; that is

$$\tilde{g} = \frac{1}{\rho_L}\left|\frac{dp}{dz}\right| + g \qquad (7)$$

for downflow, and

$$\tilde{g} = \frac{1}{\rho_L}\left|\frac{dp}{dz}\right| - g \qquad (8)$$

for upflow.

For single-phase flow a characteristic shear stress is usually defined as τ_W. In annular flow, the choice is not so straightforward, since τ_W can approach zero or even be negative and since there is an imposed shear stress at the interface τ_i. Hughmark (1973) suggested using a characteristic shear stress τ_c which is an arithmetic mean of τ_i and τ_W.

The characteristic shear used in this paper is a weighted average of τ_W and τ_i:

$$\tau_c = \tau_W\left(1 - \frac{2}{3}\frac{m}{d_t}\right) - \frac{1}{3}\rho_L \tilde{g} m \qquad (9)$$

which, for $m/d_t \to 0$, is given as

$$\tau_c = \frac{2}{3}\tau_W + \frac{1}{3}\tau_i \qquad (10)$$

This choice has the advantage that in the range of m/d_t usually encountered in annular flow, the effect of a parameter which characterizes shear variation in the liquid is minimized. This simplified the relation between the dimensionless height and the Reynolds number in the many cases where the effect of this parameter αm^- can be neglected.

A dimensionless form of the force balance can be written as

$$\tau^+ = \frac{R^+}{R^+ - y^+}\left[\frac{1 - \frac{1}{3}\alpha m^+}{1 - \frac{1}{3}\frac{m^+}{R^+}}\right]$$

$$+ \alpha m^+ \left(\frac{y^+}{m^+}\right) \left[\frac{1 - \frac{y^+}{2R^+}}{1 - \frac{y^+}{R^+}} \right] \quad (11)$$

where τ has been normalized with respect to τ_c and m, R, y, with respect to the friction velocity

$$u^* = \left(\frac{\tau_c}{\rho_L}\right)^{1/2}$$

and the kinematic viscosity ν_L. The parameter α is defined as

$$\alpha = -\frac{\tilde{g}\nu_L}{u^{*3}} \quad (12)$$

so that

$$\alpha m^+ = -\frac{\tilde{g}m}{u^{*2}} \quad (13)$$

The shear stress distribution in the liquid is related to the velocity gradient by using an eddy viscosity concept

$$\tau^+ = \left(1 + \frac{\epsilon}{\nu_L}\right)\frac{du^+}{dy^+} \quad (14)$$

The variation of ϵ/ν_L is obtained from the van Driest mixing length relation derived for single-phase flows:

$$l = \kappa y \left[1 - \exp\left(-\frac{y^+\tau^{+1/2}}{A}\right)\right] \quad (15)$$

$$\frac{\epsilon}{\nu_L} = \frac{l^2 u^{*2}}{\nu_L^2} \left|\frac{du^+}{dy^+}\right| \quad (16)$$

The parameter A, which characterizes the thickness of the viscous wall region, was given by van Driest as

$$A \cong 26 \quad (17)$$

Kays (1970) has argued that A should be a function of the pressure gradient. From the empirical relation presented by him, we get

$$A \cong 26 + 65\,000(\alpha m^+)^2 \quad (18)$$

Use of the above relation for eddy viscosity implies that the eddy viscosity profile in a highly sheared thin film with waves is the same as that for single-phase flows. The validity of this fundamental assumption will be demonstrated a posteriori by the degree of success in correlating experimental data.

Equation (14) can be integrated to get the velocity distribution in the wall layer by using Equation (11) for the stress distribution and (15), (16), (17), or (18) to calculate the variation of eddy viscosity. The volumetric flow rate in the wall layer Q can then be calculated from this velocity distribution. From such a calculation, the dimensionless film height will have the following functionality:

$$m^+ = f\left(Re_{LF}, \alpha m^+, \frac{m^+}{R^+}\right), \quad (19)$$

where $Re_{LF} = 4Q/P\nu_L$, and P is the tube perimeter.

Solutions for Small m^+/R^+

It is found that over the range of m^+/R^+ of interest in gas-liquid flows the influence of this parameter can be neglected, so that the solutions are the same as for the flow of a layer along a flat wall. This is illustrated by the calculated results shown in Figure 1. These were obtained with $\kappa = 0.40$ for the case of $\alpha m^+ = 0$.

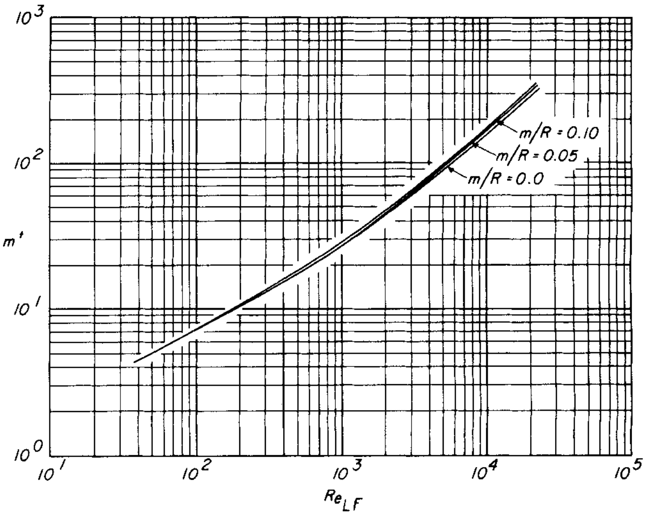

Fig. 1. Effect of radius on thickness relation.

Consequently, solutions are developed for $m^+/R^+ \to 0$. In this limit, the characteristic shear is given as $\tau_c = \tau_W - 1/3\, \rho_L \tilde{g} m$. The dimensionless shear distribution is

$$\tau^+ = 1 - \frac{1}{3}(\alpha m^+) + \alpha m^+ \left(\frac{y^+}{m^+}\right) \quad (20)$$

and the parameter αm^+ is given as

$$\alpha m^+ = \frac{-1}{\frac{\tau_i}{\rho_L \tilde{g} m} + \frac{2}{3}} \quad (21)$$

or as

$$\alpha m^+ = \frac{-1}{\frac{\tau_W}{\rho_L \tilde{g} m} - \frac{1}{3}} \quad (22)$$

The values of αm^+ for several limiting cases can be seen immediately. For a high velocity gas core, τ_i is very large and $\alpha m^+ \cong 0$. Also, in horizontal flow

$$\frac{\tau_i}{\rho_L \tilde{g} m} \simeq \frac{\frac{d_t}{4}\frac{dp}{dz}}{m\frac{dp}{dz}}$$

is a large number and $\alpha m^+ \cong 0$. Thus, αm^+ may be expected to be near zero for the type of flows usually existing in the annular regime. For free falling layers, $\tau_i = 0$ and $\alpha m^+ = -3/2$. For a very low gas velocity in vertical upflow, the shear at the wall can be small relative to τ_i and will be zero when $\alpha m^+ = 3$. Thus, the range of αm^+ is $+3$ to $-3/2$, with positive values corresponding to upflow and negative values to downflow.

For laminar flows

$$\tau^+ = \frac{du^+}{dy^+}$$

so that

$$\frac{du^+}{dy^+} = 1 - \frac{1}{3}(\alpha m^+) + \alpha m^+\left(\frac{y^+}{m^+}\right) \quad (23)$$

The volumetric flow relation calculated from (23) is

$$Re = 2m^{+2} \quad (24)$$

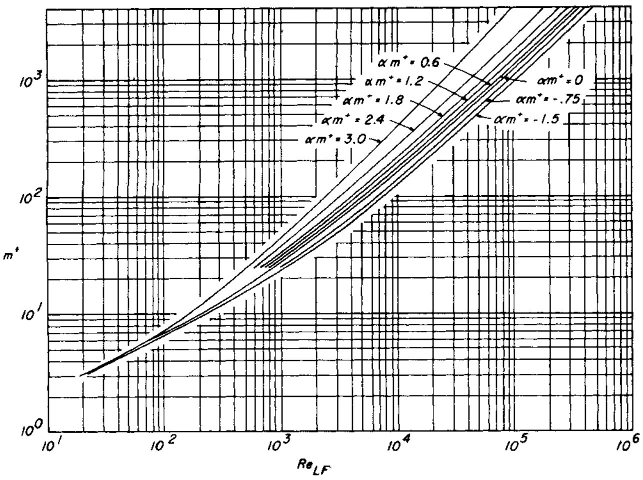

Fig. 2. Dimensionless wall layer thickness, $A = 26$.

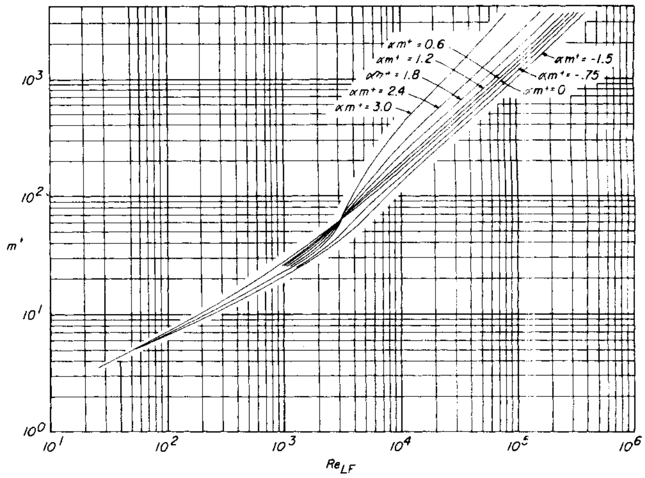

Fig. 3. Dimensionless wall layer thickness, $A = 26 + 65\,000\,(\alpha m^+)^2$.

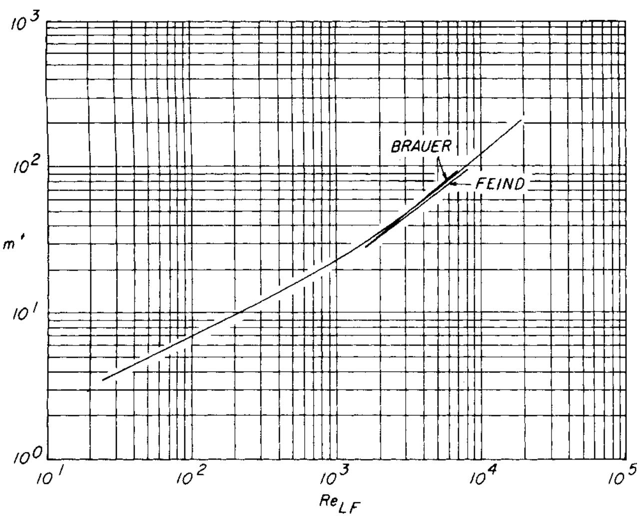

Fig. 4. Comparison of calculated thickness relation with correlations for free falling layers.

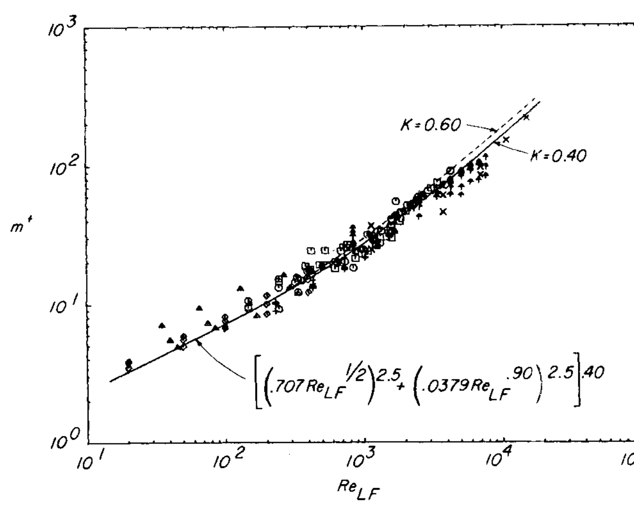

Fig. 5. Comparison of calculated thickness relation with data for vertical annular flows. (See Table 1 for key).

or

$$m^+ = 0.707\, Re^{0.5}$$

It is noted that the dependency on αm^+ does not appear in this relation. This is a consequence of choosing a characteristic stress defined by Equation (9).

For extremely large Re_{LF}, a friction factor for the liquid flow can be defined which is the same as for single-phase flows:

$$\frac{\tau_W}{\frac{1}{2}\rho_L u_L^2} = 0.046\, Re_L^{-0.20} \tag{25}$$

and the characteristic stress is given as

$$\tau_c = \frac{\tau_W}{1 - \frac{1}{3}\alpha m^+} \tag{26}$$

Then, the dimensionless height of the wall layer is

$$m^+ = \frac{m\sqrt{\frac{\tau_c}{\rho_L}}}{\nu_L}$$

$$= \frac{m\sqrt{\frac{\tau_W}{\left(1 - \frac{1}{3}\alpha m^+\right)\rho_L}}}{\nu_L}$$

$$= \frac{m u_L}{\nu_L}\sqrt{\frac{0.023}{Re_{LF}^{0.20}}}\sqrt{\frac{1}{1 - \frac{1}{3}\alpha m^+}}$$

$$= \frac{Re_{LF}}{4}\,(0.152\, Re^{-0.10})\sqrt{\frac{1}{1 - \frac{1}{3}\alpha m^+}}$$

$$= \frac{0.0379\, Re_{LF}^{0.90}}{\sqrt{1 - \frac{1}{3}\alpha m^+}} \tag{27}$$

For $\alpha m^+ = 0$, this is simply

$$m^+ = 0.0379\, Re_{LF}^{0.90} \tag{28}$$

This computation of the asymptotic behavior for large Re_{LF} is limited to a range of αm^+ where the single-phase friction factor relation might be expected to hold. For $\tau_W = 0$ this is not the case, and, in fact, (27) gives an indeterminate form for the relation between m^+ and Re_{LF}.

It is to be noted that neither (27) nor the equation for laminar flow includes A. Thus, the influence of this parameter should be limited to an intermediate range of liquid Reynolds numbers.

Computed values of m^+ are plotted in Figures 2 and 3 using the van Driest relation for eddy viscosity with $\kappa = 0.40$. These two figures correspond to cases for which the constant A in the van Driest relation is given by

(17) and (18), respectively. At small enough Re_{LF}, the height relation follows Equation (24) for laminar flow; at larger Re_{LF}, it follows Equation (27) for $\alpha m^+ < 2.4$. As expected, the effect of A is limited to a transition region.

Comparison with Measurements

For free falling films, $\tau_i = 0$ and $\alpha m^+ = -3/2$. In Figure 4, the calculated relationship between m^+ and Re_{LF} with $A = 26$ is shown for this case. For comparison, the measurements made by Brauer and Feind for turbulent free falling films are also shown. Good agreement is noted

The calculated curve can be represented by

$$m^+ = [(0.707\, Re^{0.5})^5 + (0.031\, Re^{0.90})^5]^{1/5} \quad (29)$$

over its entire range. Note that for free falling films

$$m^+ = \frac{m^{3/2}\sqrt{\frac{2}{3}g}}{\nu_L} \quad (30)$$

since

$$\tau_c = \sqrt{\frac{2}{3}\rho_L g m} \quad (31)$$

For gas-liquid flows, αm^+ is not fixed as for free falling films but can vary between 0 and 3 for upflow and 0 and $-3/2$ for downflow. Figure 2 shows the calculated relation between m^+ and Re_{LF} for different values of αm^+, with $A = 26$. The possible variation is seen to be fairly large.

However, for the vertical gas-liquid flow data summarized in Table 1, virtually all the experimental values of αm^+ are considerably less than their possible maxima. About 90% of the data points are less than half of the maximum, and only a few approach it. In fact, as shown in Figure 5, the data are represented quite well by the calculated curve for $\alpha m^+ = 0$. (The key to the symbols used for the data points is given in Table 1.) A closer examination of the data has not revealed any consistent influence of αm^+, and it is not presently possible to distinguish whether Equation (17) or (18) is the better one to use.

Therefore, the available data for gas-liquid vertical flows are represented reasonably well by the following empirical fit to the calculated relation for $\alpha m^+ = 0$.

$$m^+ = [(0.707\, Re_{LF}^{0.5})^{2.5} + (0.0379\, Re_{LF}^{0.90})^{2.5}]^{0.40} \quad (32)$$

This relation is simply an empirical matching of the asymptotes given in Equations (24) and (27) for $\alpha m^+ = 0$. Neither the form nor the indexes have any special significance. Equation (29) is a similar result.

It has been suggested that for thin turbulent wall layers, the von Karman constant should be taken as 0.60 rather than its usual value of 0.40 (Miya, 1970). The dotted line in Figure 5 shows the calculated height relationship for $\alpha m^+ = 0$ with $\kappa = 0.60$. The change is not great, and it is difficult at this time to say that value different from $\kappa = 0.40$ would produce a better correlation.

For horizontal gas-liquid flows, $\alpha m^+ \cong 0$. A comparison of the calculations with a circumferential average of the height measurements of Swanson and of Butterworth is given in Figure 6. The spread of the data about the calculated curve is about the same as for vertical flows. Until more data are available, it would appear that (32) should be used to approximate the heights

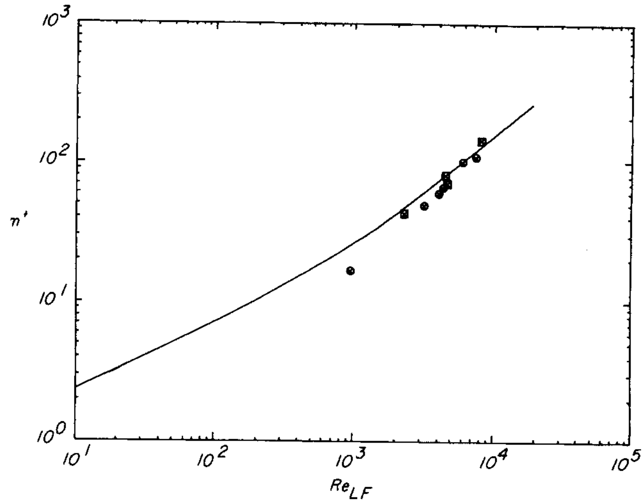

Fig. 6. Comparison of calculated thickness relation with data for horizontal annular flows. (See Table 1 for key).

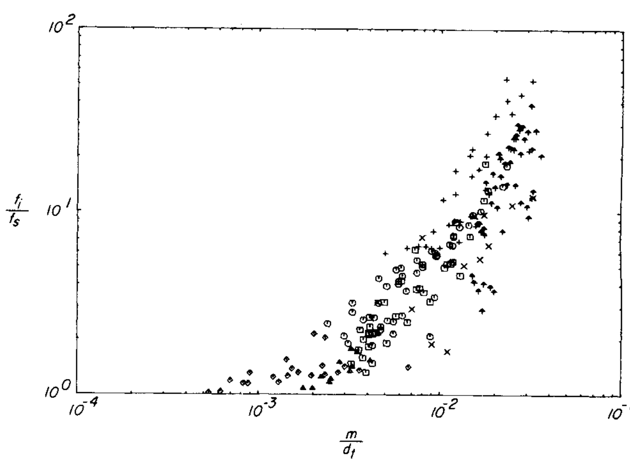

Fig. 7. Friction factor as a function of wall-layer thickness. (See Table 1 for key).

of the wall layer for horizontal as well as for vertical gas-liquid flows.

RELATION FOR THE INTERFACIAL STRESS

Development of an Equation for τ_i

As mentioned in the introduction, the work of a number of previous investigators (Hewitt and Hall-Taylor, 1970, p. 87) would suggest that the ratio of the interfacial stress to that for a smooth wall can be correlated by assuming it is primarily a function of the ratio of the height of the wall layer to the diameter of the tube

$$\frac{\tau_i}{\tau_s} = \frac{f_i}{f_s} = f\left(\frac{m}{d_t}\right) \quad (33)$$

This type of approach is illustrated in Figure 7, where the interfacial friction factor is based on the superfacial gas velocity

$$f_i = \frac{\tau_i}{\frac{1}{2}\rho_G u_G^2} \quad (34)$$

and the friction factor for a smooth surface has been calculated from

$$f_s = 0.046\, Re_G^{-0.20} \quad (35)$$

To be strictly consistent with the model of a roughened tube, the height of the film should be subtracted from the radius when the gas velocity is computed, and then this should be taken relative to the surface velocity of the liquid. However, this approach introduces tremendous difficulties when eliminating variables later, and it has

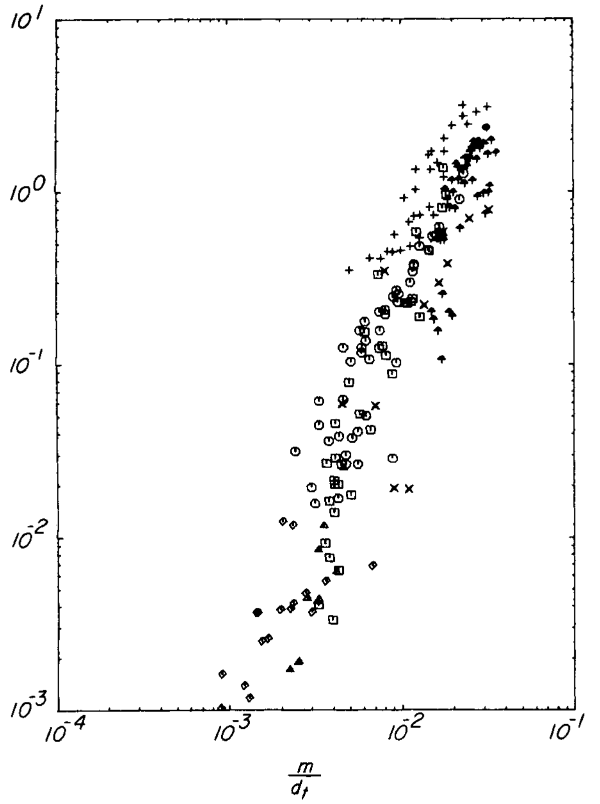

Fig. 8. Effective roughness as a function of wall-layer thickness. (See Table 1 for key).

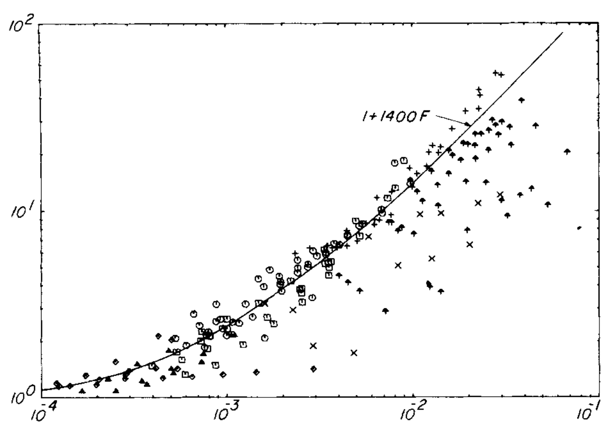

Fig. 9. Friction factor as a function of F, vertical flows. (See Table 1 for key).

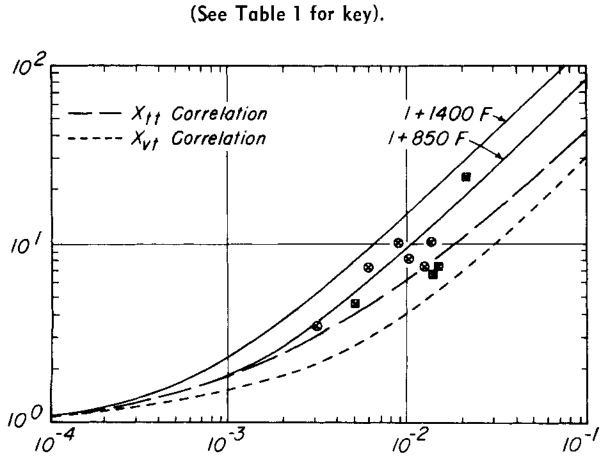

Fig. 10. Friction factor as a function of F, horizontal flows. (See Table 1 for key).

relatively little effect on the friction factors. Consequently, it has not been used.

Another method of correlating the data, shown in Figure 8, would be to plot equivalent sand roughness against m/d_t, where the sand roughness is calculated from Equation (20.35*a*) given by Schlichting (1968).

One of the difficulties with the methods of correlation shown in Figures 7 and 8 is that both the ordinate and the abscissa are strong functions of the variable that is being correlated. A superior approach can be developed by using the relations developed in the previous section to eliminate the dependency of the abscissa on τ_i.

For gas-liquid flows, it has been found that

$$m^+ = \gamma(Re_{LF}) \tag{36}$$

where $\gamma(Re)$ is the right-hand side of Equation (32). From the definition of m^+ and Equation (35) for f_s, it follows that

$$\frac{m}{d_t} = \frac{6.59F\sqrt{\frac{\tau_i}{\tau_c}}}{\sqrt{\frac{f_i}{f_s}}} \tag{37}$$

where

$$F \equiv \frac{\gamma(Re_{LF})}{Re_G^{0.90}}\frac{\nu_L}{\nu_G}\sqrt{\frac{\rho_L}{\rho_G}} \tag{38}$$

From the definition of τ_c, it can be shown that

$$\frac{\tau_i}{\tau_c} = \sqrt{1\frac{2}{3}\alpha m^+} \tag{39}$$

Consequently, an assumption of the type made in (33) suggests that f_i/f_s is strongly dependent on the function F and only weakly dependent on αm^+:

$$\frac{f_i}{f_s} = g(F, \alpha m^+) \tag{40}$$

Measured values of f_i/f_s are plotted against F in Figure 9 for vertical flows and in Figure 10 for horizontal flows. By using F as the independent variable, the abscissa is expressed entirely in terms of controlled variables.

Figure 9 confirms the suggested strong dependency on F. The bulk of the data can be represented by the relation

$$\frac{f_i}{f_s} - 1 = 1\,400\,F \tag{41}$$

shown on the figure. However, while the upflow data correlate quite well, there is much more scatter in the downflow data. Most of these data points fall well below the line represented by Equation (41).

A closer examination of the data in Figure 9 showed that the points with the largest deviation tended to have the lowest values of Re_G, although the effect was not a simple one. By trial and error, it was found that there is a dependence on a closely related group $\tau_i/\rho gm$. Figure 11 is a plot of

$$\frac{\frac{f_i}{f_s} - 1}{1\,400\,F}$$

or the ratio of measured to predicted friction factors, against $|\tau_i/\rho_L gm|$. There is seen to be a strong dependence on $\tau_i/\rho_L gm$ at low values of the parameter but little dependence at high values.

In upflow, τ_i is necessarily very high to maintain annular flow; however, in downflow, τ_i can be very low or even zero. Thus the upflow data do not show the influence of this group and correlate much better with F than do the downflow data.

In downflows at zero gas velocity, the interface of the free falling liquid layer is roughened by waves caused by gravitational force $\rho_L gm$. As the gas flow is increased, the wave structure changes until the wave shape is characterized only by the drag force of the gas at the interface τ_i. The influence of $\tau_i/\rho_L gm$ can therefore be interpreted as due to a change of the characteristics of the wave pattern. This is another reason why the height of the wall layer is not an appropriate variable for the correlation of friction factors.

The effect of the parameter $\tau_i/\rho_L gm$ can be eliminated empirically by replacing F with

$$F\left[1-\exp\left(-\left|\frac{\tau_i}{\rho_L gm}\right|\right)\right] \tag{42}$$

A plot of f_i/f_s against this group is shown in Figure 12, and the data are seen to collapse fairly well.

The parameter $\tau_i/\rho_L gm$, although physically meaningful, is not convenient to use since it is not in terms of controlled variables. However, it can be rewritten as

$$\frac{\tau_i}{\rho_L gm} = \frac{\frac{1}{2}\rho_G u_G^2 f_s}{\rho_L g d_t}\,\frac{f_i/f_s}{m/d_t} \tag{43}$$

$$= \frac{1}{2G}\,\frac{f_i/f_s}{m/d_t}$$

where

$$G \equiv \frac{\rho_L g d_t}{\rho_G u_G^2 f_s} \tag{44}$$

If we use the relations developed above for f_i/f_s and m/d_t

$$\frac{\tau_i}{\rho_L gm} = \frac{1}{G}\,\frac{\left(1+1\,400F\left[1-\exp\left(-\left|\frac{\tau_i}{\rho_L gm}\right|\right)\right]\right)^{3/2}}{13.2F}$$

$$\simeq \frac{1}{G}\,\frac{(1+1\,400F)^{3/2}}{13.2F} \tag{45}$$

Figure 13 shows the friction factor data, using Equation (45) to eliminate $\tau_i/\rho_L gm$. It does not do as good a job of collapsing the data as Equation (42). However, it is much more convenient to use because it is entirely in terms of controlled variables.

It has been suggested that friction factors might also depend on the amount of entrained flow because of their effect on the gas core turbulence or their contribution to τ_i due to momentum transfer associated with the exchange of liquid between the core and the wall layer. These effects should increase with increasing entrainment. In order to explore this influence of entrainment, the plot of $(f_i/f_s)/(1 + 1\,400F)$ against (W_{LE}/W_G) shown in Figure 14 was constructed. No noticeable influence of (W_{LE}/W_G) is indicated. The gas Reynolds number was also tested in this same way, and again no influence was noted.

Consequently, it has been concluded that any spread of the data in Figure 12 represents errors or uncontrolled extraneous effects and that the best correlation of friction factors for vertical annular flows is

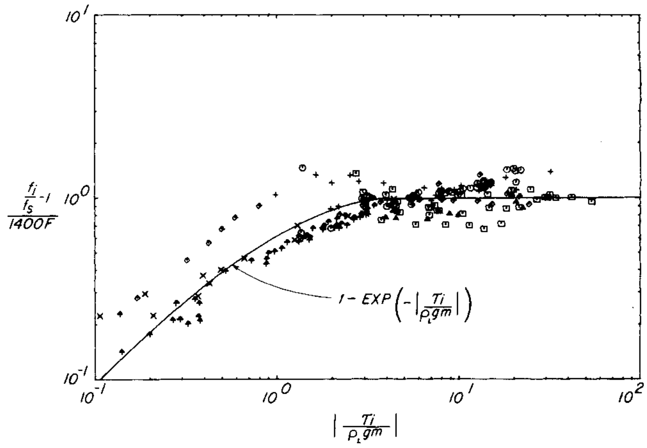

Fig. 11. Effect of interfacial shear on friction factors, vertical flows. (See Table 1 for key).

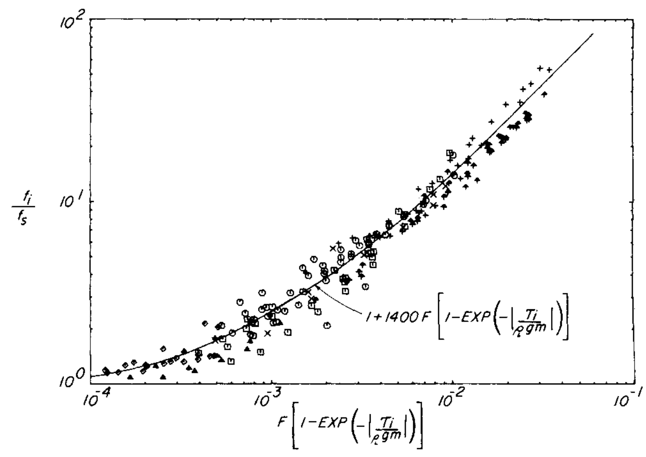

Fig. 12. Final form of friction factor correlation, vertical flows. (See Table 1 for key).

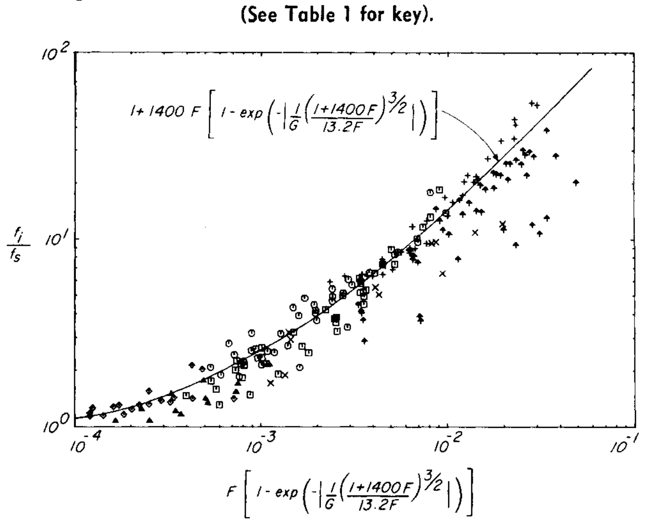

Fig. 13. Approximate friction factor correlation, downflows. (See Table 1 for key).

$$\frac{f_i}{f_s} = 1 + 1\,400F\left[1 - \exp\left(-\left|\frac{1}{G}\,\frac{(1+1\,400F)^{3/2}}{13.2F}\right|\right)\right] \tag{46a}$$

For horizontal flows, Equation (46a) reduces to (41). The comparison of the limited data available for horizontal flows in Figure (10) suggests that it can overpredict the value of $(f_i/f_s) - 1$ by as much as 150%. Consequently, we would tentatively suggest that the following relation be used for horizontal flows:

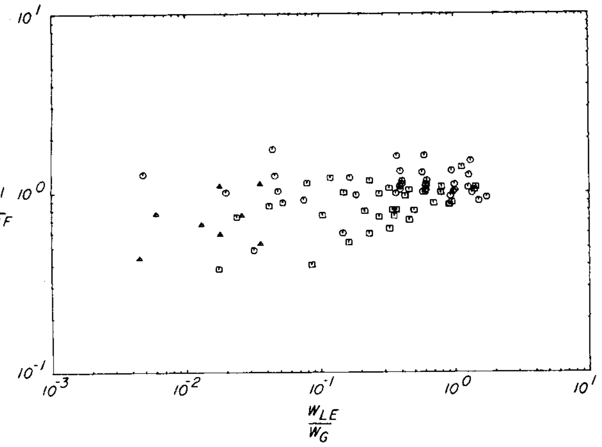

Fig. 14. Effect of entrained flow on friction factors. (See Table 1 for key).

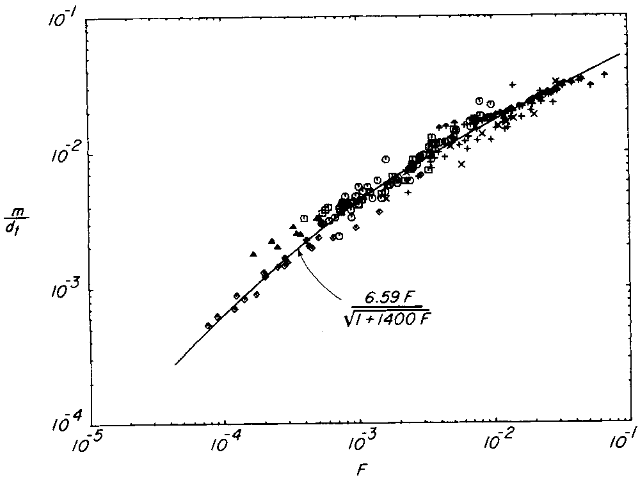

Fig. 15. Wall-layer thickness as a function of *F*, vertical flows. (See Table 1 for key).

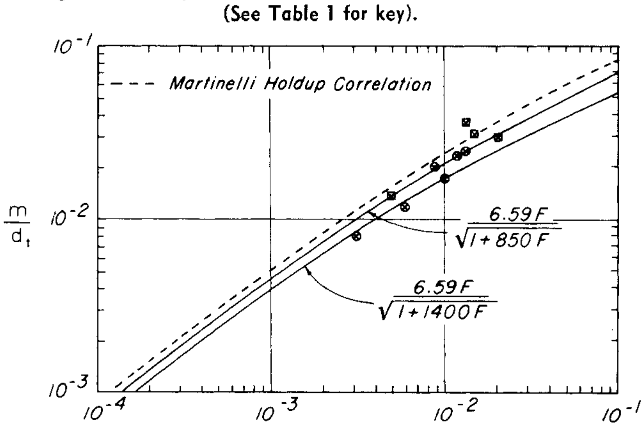

Fig. 16. Wall-layer thickness as a function of *F*, horizontal flows. (See Table 1 for key).

$$\frac{f_i}{f_s} = 1 + 850F \qquad (46b)$$

This is shown in Figure 10 as another solid line.

RELATION FOR THE HEIGHT OF THE WALL LAYER IN TERMS OF CONTROLLED VARIABLES

Equations (29) and (30) give the height of free falling wall layers in terms of controlled variables and consequently are quite easy to use. This is not the case if Figure 1 or 3 is to be used for gas-liquid flows. In order to calculate m from m^+, one needs the value of τ_c or of τ_i, because

$$\tau_c = \frac{\tau_c}{\tau_i}\tau_i$$

$$= \tau_i\left(1 + \frac{2}{3}\alpha m^+\right) \qquad (47)$$

The iterative procedure to be used for gas-liquid flow would be to calculate first τ_i from (46*a*). A value for αm^+ is guessed, and τ_c is calculated from (39). The height m can then be obtained from Figure 2 or 3. A new estimate for αm^+ is obtained from Equation (13), and the procedure repeated.

A more direct approximate method is available if it is assumed that the height of the wall layer for gas-liquid flows can be calculated reasonably accurately by assuming $\alpha m^+ = 0$. Under these circumstances, we obtain from (37) and (46*a*)

$$\frac{m}{d_t} = \frac{6.59F}{\left(1 + 1\,400F\left[1 - \exp\left(-\left|\frac{1}{G}\frac{(1 + 1\,400F)^{3/2}}{13.2F}\right|\right)\right]\right)^{1/2}} \qquad (48a)$$

This expression is rather insensitive to G and can be approximated by

$$\frac{m}{d_t} = \frac{6.59F}{(1 + 1\,400F)^{1/2}} \qquad (48b)$$

A plot of measured values of m/d_t against F is shown in Figure 15. It is seen that (48*b*) is a good approximation of presently available measurements for vertical gas-liquid flows.

A similar expression is obtained for horizontal flows by combining Equation (37) and (46*b*):

$$\frac{m}{d_t} = \frac{6.59F}{(1 + 850F)^{1/2}} \qquad (48c)$$

This is compared in Figure 16, with the measured values of m/d_t for horizontal flow. Agreement is not quite so good as for the vertical case.

Comparison with Martinelli's Correlation

There are strong similarities between the method of correlation presented here and that developed by Martinelli. If W_{LF} is used in place of W_L in the Martinelli flow parameters X_{vt} and X_{tt}, the parameter F becomes directly proportional to X_{vt} at low Re_{LF}

$$F = \frac{1}{\sqrt{2}\,Re_G^{0.4}}\,\frac{Re_{LF}^{0.5}}{Re_G^{0.5}}\,\frac{\mu_L}{\mu_G}\,\frac{\rho_G^{0.5}}{\rho_L^{0.5}}$$

$$= 0.0379\,X_{vt} \qquad (49a)$$

and directly proportional to X_{tt} at large Re_{LF}:

$$F = \frac{0.0379\,Re_{LF}^{0.90}}{Re_G^{0.90}}\,\frac{\mu_L}{\mu_G}\,\frac{\rho_G^{0.5}}{\rho_L^{0.5}}$$

$$= 0.0379\,X_{tt} \qquad (49b)$$

Thus, F may be thought of as a generalized Martinelli parameter for turbulent gas flow using W_{LF} instead of W_L.

The relations for wall-layer thickness and friction factor developed here can now be compared to Martinelli's work. The wall-layer thickness relation given in Equation (2) becomes

$$\frac{4\dfrac{m}{d_t}}{1 - 4\dfrac{m}{d_t}} = 0.28F^{0.71} \qquad (50)$$

This is drawn in Figure 16 as a dashed line for comparison with Equation (48*b*). Similarly, Martinelli's friction factor results are shown as a function of F in Figure 10 for comparison with Equation (41). The shapes of the curves are similar in both cases, but there are large numerical differences.

It might be thought that because Martinelli's data were for horizontal flow, his results would better fit the horizontal flow data presented here. However, any better agreement is marginal, and it is difficult to make a choice on the basis of the small amount of data available.

ACKNOWLEDGMENT

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NOTATION

A = parameter in eddy viscosity relation
A_t = cross-sectional area of the channel
d_c = diameter of the gas core
d_t = hydraulic diameter of the channel
D = diffusivity
E = fraction of liquid entrained, $E = W_{LE}/W_L$
f_i = gas-phase friction factor based on interfacial shear, $f_i = \tau_i/\frac{1}{2}\rho_G u_G^2$
f_s = gas-phase friction factor which would exist in a smooth tube, $f_s = 0.046/Re_G^{0.20}$
F = dimensionless group containing flow rates and fluid properties, $F = \frac{\gamma(Re_{LF})}{Re_G^{0.90}} \frac{\nu_L}{\nu_G} \sqrt{\frac{\rho_L}{\rho_G}}$
g = acceleration of gravity
$\tilde{g}$ = parameter showing combined effects of gravity and pressure gradient, $\tilde{g} = \frac{1}{\rho_L}\left|\frac{dp}{dz}\right| - g$
G = dimensionless group characterizing gravity effects, $G = \rho_L g d_t/\rho_G u_G^2 f_s$
l = mixing length
m = thickness of the wall layer
m^+ = dimensionless film thickness, $m^+ = mu^*/\nu_L$
P = wetted perimeter of the channel
Q = volumetric flow rate
r = radial distance
r^+ = dimensionless radial distance, $r^+ = ru^*/\nu_L$
R = tube radius
R = dimensionless tube radius, $R^+ = Ru^*/\nu_L$
Re_G = gas Reynolds number calculated as if the gas filled the whole tube $Re_G = d_t W_G/A_t \mu_G$
Re_{LF} = Reynolds number of the liquid flowing in the wall layer $Re_{LF} = 4W_{LF}/\mu_L P$
u^* = friction velocity, $u^* = \sqrt{\tau_c/\rho_L}$
u_G = superficial gas velocity
u_L = average velocity in the liquid film
W_G = mass flow rate of the gas in the units of mass per unit time
W_L = mass flow rate of the liquid in the units of mass per unit time
W_{LE} = mass flow rate of entrained liquid
W_{LF} = mass flow rate of the liquid in the wall layer
X, X_{tt}, X_{vt} = Martinelli parameter,

$$X_{vt} = 16.9\left(\frac{\mu_L}{\mu_G}\right)^{0.50}\left(\frac{\rho_G}{\rho_L}\right)^{0.50}\left(\frac{W_L}{W_G}\right)^{0.50} Re_G^{-0.40}$$

$$X_{tt} = \left(\frac{\mu_L}{\mu_G}\right)^{0.10}\left(\frac{\rho_G}{\rho_L}\right)^{0.50}\left(\frac{W_L}{W_G}\right)^{0.90}$$

y = distance from wall, $R - r$
y^+ = dimensionless distance from wall, $y^+ = yu^*/\nu$
dp_F/dz = frictional pressure gradient

Greek Letters

α = parameter characterizing shear variation in film, $\alpha = -\tilde{g}\nu_L/u^{*3}$
β = void fraction of the liquid
ϵ = eddy viscosity
γ = function of liquid film Reynolds number, $\gamma = [(0.707\, Re_{LF}^{1/2})^{2.5} + (0.0379\, Re_{LF}^{0.90})^{2.5}]^{0.40}$
κ = van Karman constant
ρ_G = gas density
ρ_L = liquid density
μ_G = gas viscosity
μ_L = liquid viscosity
ν_G = gas kinematic viscosity in units of length squared over time
ν_L = liquid kinematic viscosity
τ^+ = dimensionless shear stress, $\tau^+ = \tau/\tau_c$
τ_c = characteristic shear stress
τ_i = interfacial shear stress
τ_s = shear stress which would exist in a smooth tube
τ_W = wall shear stress

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Mass Transfer of Dissolved Gases Through Tubular Membrane

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Mass transfer characteristics are studied theoretically and experimentally for dissolved oxygen and carbon dioxide in fully developed laminar flow through a gas permeable membrane tubing. The gas transfer rate is correlated with the liquid flow rate and tubing size. Membrane resistance together with liquid phase resistance determine the overall mass transport rate.

SCOPE

In recent years, polymer membranes have been employed in industry as media for the separation or purification of mixtures which are otherwise difficult to separate economically (Lacey and Loeb, 1972; Hwang and Kammermeyer, 1975). Membrane processes such as dialysis, electrodialysis, reverse osmosis, and gas separation are often limited by slow solute transport in the liquid phase.

Liquid phase resistance to dissolved gas permeation was not recognized in earlier work (Yasuda, 1967). Later, the importance of liquid boundary resistance was reported by Hwang et al. (1971) in their study of the effect of membrane thickness on permeability. Recently, however, liquid resistance has been considered as the rate controlling resistance, and membrane resistance is usually ignored. In this paper both membrane and liquid resistances are taken into consideration to treat the general case, and the significance of membrane phase resistance relative to the liquid phase resistance is studied. Water containing low oxygen or high carbon dioxide content was passed through silicone rubber membrane tubing. Oxygen in the atmosphere permeates through the membrane into water, and the dissolved carbon dioxide permeates back into the atmosphere. The dissolved gas concentration and wall flux vary along the tube length. This boundary condition differs from the common assumption of constant concentration or constant wall flux.

The solution to this problem reveals the significance of membrane resistance to the gas transfer under various operating conditions. It provides information as to when membrane resistance can and cannot be neglected. The results developed in this paper can be applied to predict the performance of other membrane processes and to evaluate transport parameters, such as membrane permeability and solute diffusivity.

CONCLUSIONS AND SIGNIFICANCE

Permeation rates of dissolved oxygen and carbon dioxide in laminar water flow are measured under various experimental conditions. The theoretical concentration profile of dissolved gas is obtained by numerical solution for a general case, in which membrane resistance is not negligible compared with the liquid phase resistance. The changes in cup mixing concentration are correlated with the dimensionless tube length X and the system parameter E. The agreement between theoretical values and experimental data is shown to be satisfactory for many different experimental conditions. The upper bound is calculated for the change of cup mixing concentration by neglecting membrane resistance. It is found that the membrane resistance becomes a significant part of the overall resistance to the dissolved gas permeation when $E < 30$ and/or $X < 0.01$. For practical application and convenience, a performance coefficient ϕ is defined as flux per unit initial pressure gradient, which can be theoretically calculated. A good agreement is achieved between the theory and experimental results.

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